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Accuracy and Limitations of the Liquid Drop Model (LDM) in Nuclear Binding Energy Calculations

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Abstract: This study aims to evaluate the accuracy of the Liquid Drop Model (LDM) in predicting atomic nuclear binding energy and binding energy per nucleon, by comparing it with reference values. LDM is based on the assumption that atomic nuclei can be treated as drops of incompressible fluid. Nuclear binding energy is calculated using the Semi-Empirical Mass Formula (SEMF), and the results are analyzed through linear regression comparison with empirical mass defect data. The calculation results show that the LDM produces small deviations for binding energy values in medium nuclei. However, this model is less accurate in predicting binding energy for light and heavy nuclei. Deviations in heavy atomic nuclei occur due to the dominant collective effect. Under these conditions, the phenomenon of atomic nuclei is more accurately explained as the interaction of all nucleons as a whole, rather than the behavior of individual nucleons. This supports the main principle of LDM in understanding heavy atomic nuclei. In addition, the calculation of binding energy per nucleon by LDM produces the highest binding energy peak in Krypton-80 with a value of 8.98 MeV/nucleon. This result differs from empirical reference values that place Iron-56 (Fe-56) as the most stable nucleus with the highest binding energy, namely 8.79 MeV/nucleon. This deviation in the stability peak highlights the limitations of LDM, particularly regarding the lack of consideration of quantum effects and nuclear shell structures that are more relevant to certain nuclei.

Keywords: Liquid drop model, mass defect, nuclear binding energy

1. Introduction

All matter in the universe is composed of atoms that have electrons and atomic nuclei. The atomic nucleus consists of basic elementary particles, namely protons and neutrons. Protons are positively charged electronic units and have a mass 1836 times greater than that of electrons. Meanwhile, neutrons are neutral particles with a mass slightly greater than that of protons. These two particles are bound together by a very strong nuclear force. Neutrons and protons bound

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together in this way are called nucleons. The atomic nucleus, consisting of protons and neutrons, is bound together by an energy known as nuclear binding energy.

Nuclear binding energy is a fundamental concept in nuclear physics that defines the stability of an atomic nucleus. Physically, binding energy is the minimum energy required to separate the nucleus into its constituent nucleons (protons and neutrons). A large total binding energy value for an atomic nucleus indicates that the nucleus is strongly bound and more stable. The calculation of nuclear binding energy (B) is based on mass defect (Δm), in accordance with Einstein's famous equation, $E = mc^2$. However, to explain how nuclear forces work collectively within the nucleus and to predict the value of binding energy theoretically, an effective descriptive model is needed, such as the Liquid Drop Model [1], [2]. This descriptive model refers to an approach or framework that simplifies the highly complex system at the core of an atom into an analogy that is easier to understand and calculate, while still providing practically accurate results.

The Liquid Drop Model (LDM) was first proposed by George Gamow in 1930 and significantly developed by Niels Bohr and John Archibald Wheeler. The LDM is based on the analogy that the atomic nucleus behaves like a drop of liquid consisting of nucleons bound by the strong nuclear force [3], [4], [5]. This analogy is effective because of two main properties of nuclei and liquids: first, the nuclear force has a short range and occurs between closest neighbors (saturation), similar to molecular bonds in liquids. Second, atomic nuclei have an almost constant density, independent of the number of nucleons (A), just like incompressible liquids. This model aims to explain the collective properties of the nucleus as a whole, ignoring the individual behavior of nucleons. The liquid drop model of the nucleus successfully expresses the property of the nucleus, namely the average energy per nucleon [6], [7].

Although it is a classical approach, LDM plays a central role in explaining two macroscopic nuclear phenomena such as fission and fusion. This model accurately explains why very heavy nuclei (such as Uranium) are unstable and tend to split (fission) due to the dominance of Coulomb repulsive forces balanced by surface tension. In addition, the LDM provides a foundation for understanding that energy is released when light nuclei merge (fusion) into nuclei with the highest binding energy per nucleon (around A=60) [3], [8]. Research conducted by Myers & Świątecki (1966) and Strutinsky (1967) shows that pure macroscopic models with LDM fail to account for shell effects, which provide extra stability at certain mass numbers [9], [10]. Although the weakness of LDM has often been found in other studies, these studies do not quantify the magnitude of the deviation in a specific mass range. Therefore, the LDM using Semi-Empirical Mass Formula (SEMF) was used in this study to analyze the binding energy of several atomic nuclei (light to heavy nuclei) compared to the binding energy in the reference by Krane (1988) [7].

2. Method

This research uses a theoretical-computational research method with a literature study and numerical simulation approach. The main focus is on the quantitative analysis of the binding energy (B) of various stable nuclides based on parameters set by the Liquid Drop Model. The primary data sources are nuclear physical constants and experimental atomic mass data sourced from the international atomic mass database (AME - Atomic Mass Evaluation). This approach allows for the calculation and comparison of theoretical nuclear binding energy with experimental data, thereby enabling the evaluation of the model's validity. The computational procedures performed in this study were carried out through the following steps.

1. Data Selection: Isotopes were selected based on mass number (A) categories (light, medium, and heavy).
2. Data Processing: Experimental binding energy was obtained from atomic mass defects using equation (2).
3. Calculation: Theoretical binding energy was calculated using Semi-Empirical Mass Formula (SEMF) parameters in equation (1).
4. Computational Tools: All calculations, linear regression analyses, and data visualizations were performed using Microsoft Excel.

The first method uses the Liquid Drop Model (LDM), which is a macroscopic approach that treats atomic nuclei as incompressible liquid droplets. The nuclear binding energy (B) is calculated using the Semi-Empirical Mass Formula (SEMF), which breaks down the total binding energy into five main terms, each representing a different physical phenomenon in the nucleus. These terms include: volume energy (proportional to the number of nucleons, A), surface energy (correcting for the loss of binding at the surface, proportional to $A^{2/3}$), Coulomb energy (electrostatic repulsive force between protons, proportional to $\frac{z^2}{A^{1/3}}$), asymmetry energy (correcting for the imbalance in the number of protons and neutrons), and pairing energy (correcting for the stability of nuclei with even/odd numbers of nucleons). The core of this method is the use of five semi-empirical constants (a_1, a_2, a_3, a_4, a_5) that must be determined by fitting experimental data on nuclear binding energy, making this model predictive yet still empirical due to these constants [7], [11]. The nuclear binding energy equation with LDM is written as in equation (1) below.

$$B(A,Z) = a_1 A - a_2 A^{\frac{2}{3}} - a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_4 \frac{(A-2Z)^2}{A} - a_5 A^{-3/4} \quad (1)$$

with $B(A,Z)$ = nuclear binding energy, A = mass number, Z = atomic number, and the values of constants a_1, a_2, a_3, a_4 and a_5 are as follows.

$a_1 = 15.753$ MeV

$a_2 = 17.804$ MeV

$a_3 = 0.7103$ MeV

$a_4 = 23.69$ MeV

$$a_5 = \begin{cases} 33.6\, A^{-3/4} & if\ N\ and\ Z\ even\ number \\ -33.6\, A^{-\frac{3}{4}} & if\ \ N\ and\ Z\ odd\ number \\ 0 & if\ N+Z = odd\ number \end{cases}$$

The second method is the calculation of pure experimental nuclear binding energy through the concept of Mass defect (ΔE). Binding energy (B) is defined as the energy released when nucleons (protons and neutrons) bind together to form a nucleus, which is equivalent to the difference between the total mass of the constituent components (free mass) and the mass of the nucleus formed (bound mass) [7]. Mathematically, the modified mass defect is calculated as equation (2) as follows.

$$\Delta E = Z(m_p + m_e) + Nm_n - (m_u + Zm_e) \quad (2)$$

The mass of 1 proton + the mass of 1 electron is equal to the mass of a hydrogen atom (m_H). Meanwhile, the mass of the nucleus + Z_{me} is equal to the atomic mass. Therefore, equation (2) can be rewritten as equation (3).

$$\Delta E = Zm_H + Nm_n - m_a \quad (3)$$

This method produces the most accurate nuclear binding energy values because it is based on actual nuclear mass measurements, making it the primary benchmark for evaluating the accuracy of theoretical models such as LDM [7].

To assess the quantitative relationship between LDM results and experimental values of ΔE, a simple linear regression method applied. The binding energy data predicted by LDM (B(Z,A)) can be plotted on the X-axis against the experimental binding energy data ΔE) derived from mass defects on the Y-axis for a large number of nuclei. If LDM is a perfect model, the result will be a straight line with a slope close to 1 and an intercept close to 0, with a correlation coefficient (R^2) close to 1. The deviation of the data points from the regression line (residuals) will explicitly show the prediction error of the LDM, particularly the model's failure to capture the shell effects and internal structure of the nucleus, which are implicitly captured by the mass defect measurements.

3. Results and Discussion

*3.1 Comparison o*f ΔE and B(A,Z)

Figure 1 shows the relationship between an energy quantity ΔE and the Nuclear Binding Energy B(A,Z). The curve shows a linear relationship, implying that ΔE is almost equal to the Nuclear Binding Energy B(A,Z), indicating a very strong correlation and the possibility that ΔE represents energy directly related to the process of nucleus formation or separation. However, there is a slight but significant upward deviation after B(A,Z) exceeds 1800, where the gradient of the curve becomes steeper. This change in gradient at high binding energies may indicate the presence of additional effects that arise in very large and highly bound nuclei, or it may be an anomaly related to theoretical calculations or limitations of the liquid drop model used in this study. The regression line equation shown is $\Delta E = 1.0162\, B(A,Z) - 12.933$ with $R^2 = 0.9979$, indicating that this linear model has a very high level of fit to the data (99.79%).

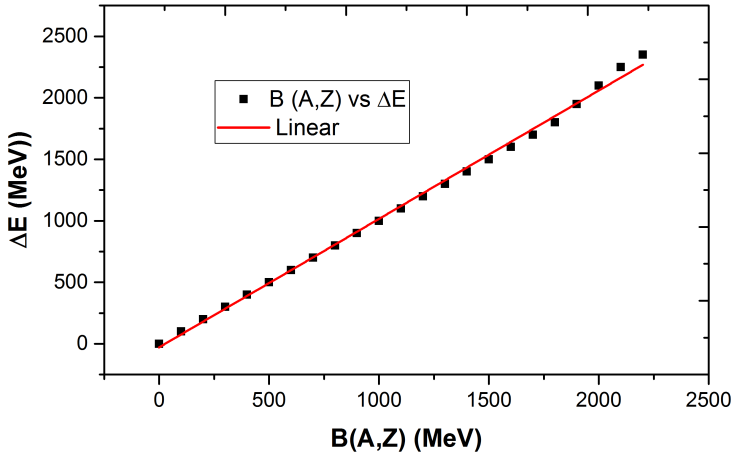

Figure 1. Comparison of ΔE and B(A,Z)

This graph shows that the calculation of nuclear binding energy can be performed using LDM for nuclei with B(A,Z) less than 1800 by using the calculation of parameters A, Z, and LDM constants (equation (1)). This calculation can be an alternative if experimental data or measurements are not available to calculate mass defects (ΔE) as in equation (3). The results of this study are consistent with other studies showing that LDM provides an excellent approach (average deviation below 1.5%) for most stable nuclides, with the largest deviation patterns occurring in light nuclei [12].

In addition to heavy nuclei (B(A,Z)> 1800), LDM does not accurately calculate the binding energy of light nuclei such as deuterium, tritium, and helium because this model is based on macroscopic assumptions of collective behavior and uniform effective density that are only valid for heavy nuclei with many nucleons. In light nuclei consisting of only two to four nucleons, these assumptions fail because there is no significant interior; almost all nucleons are on the surface, making the surface energy term artificially dominant. Furthermore, the binding energy of light nuclei is strongly influenced by individual interactions between nucleons and specific quantum mechanical effects, which LDM ignores. Therefore, light nucleus energies must be calculated using the Nuclear Shell Model or fundamental potential-based calculations rather than the collective statistical approximation of the LDM [7], [13], [14]. Table 1 shows a comparison of binding energies calculated using LDM and ΔE for the elements Hydrogen, Deuterium, Tritium, and Helium.

Table 1. Calculated nuclear binding energy (LDM) and ΔE for the light nucleus

Element	A	Z	B(A,Z) (MeV)	ΔE (MeV)	Deviation
Hydrogen	1	1	-25.741	0	23.31%
Deuterium	2	1	-16.80014265	2.224422	113.24%
Tritium	3	1	2.192652985	8.482239	286.85%
Helium	3	2	1.208385108	7.718409	538.74%

Table 2. Calculated nuclear binding energy (LDM) and ΔE for the medium nucleus

Element	A	Z	B(A,Z) (MeV)	ΔE (MeV)	Deviation
Antimony	121	51	1027.830099	1026.33322	0.15%
	123	51	1042.182558	1042.105378	0.01%
Tellurium	120	52	1019.942516	1017.29115	0.26%

	122	52	1036.25601	1034.340395	0.18%
	123	52	1043.97884	1041.269823	0.26%
	124	52	1051.421762	1050.695672	0.07%
	125	52	1058.591521	1057.27113	0.12%
	126	52	1065.49465	1066.384926	0.08%
	128	52	1078.526126	1081.451939	0.27%

Table 3. Calculated nuclear binding energy (LDM) and ΔE for the heavy nucleus

Element	A	Z	B(A,Z) (MeV)	ΔE (MeV)	Deviation
Dubnium	262	105	1932.468975	2021.90738	4.63%
Seaborgium	266	106	1956.198382	2052.664578	4.93%
Bohrium	264	107	1936.851159	2035.498896	5.09%
Hassium	269	108	1967.993291	2085.464556	5.97%
Meitnerium	268	109	1954.724393	2076.566868	6.23%
Ununnilium	272	110	1978.767098	2108.000335	6.53%
Unununium	272	111	1972.004887	2107.150808	6.85%
Ununbium	277	112	2003.406811	2148.155437	7.23%
Ununquadium	289	114	2076.244394	2243.447887	8.05%
Ununhexium	289	116	2065.915993	2241.882968	8.52%
Ununoctium	293	118	2082.114048	2272.603837	9.15%

The data in Tables 2 and 3 show that the deviation between calculations using LDM and ΔE is low (below 1% for medium nuclei such as Antimony/Sb and Tellurium/Te, and less than 10% for heavy nuclei Z=105-118). Although this 10% value is statistically higher than the deviations found in heavy nuclei, it is physically consistent with LDM limitations. This is because these nuclei satisfy the main macroscopic assumptions of the LDM model. The LDM assumes that nuclei behave like incompressible liquid droplets, where the binding energy behavior is dominated by the collective interaction of many nucleons. For medium nuclei ($A \approx 120$), the number of nucleons is large enough that the collective effect is more prominent than the behavior of individual nucleons. In addition, the volume-to-surface ratio is higher than in light nuclei, so that the volume energy term becomes dominant. In heavy nuclei ($Z = 105 - 118$), although the Coulomb force that pushes the nuclei apart is very large, the very high number of nucleons ($A > 280$) greatly strengthens the validity of the collective assumption. In heavy nuclei, the very large number of nucleons (A) reinforces the validity of the collective assumption. However, this macroscopic description has limitations because it does not take into account shell effects and nuclear deformation, which become increasingly prominent in heavy elements.

3.2 Binding Energy per Nucleon (B(Z,A)/A)

Figure 2 shows that for small A (light nuclei), the predicted value of ($B(Z,A)/A$) is very low (even negative at A=1), which is inconsistent with the reference. This indicates that LDM fails to capture the specific interactions of individual nucleons and effects related to quantum symmetry. Meanwhile, the value of ($B(Z,A)/A$) increases sharply as A increases to around 20. This increase is dominated by the volume term, which increases rapidly compared to the surface term and the Coulomb term. This curve saturates at around A = 20, indicating the fundamental

nature of saturated nuclear force, which means that each nucleon interacts only with a small number of its closest neighbors. This can also be seen in the R^2 in Figure 1, where the light nucleus is lower than the medium nucleus, because the variance of the experimental data is not only influenced by volume and surface area but also by the quantum configuration of the nucleons. This calculated data contrasts with the experimental results for the binding energy per nucleon. This graph generally shows that LDM successfully captures the macroscopic trend of nuclear binding energy (the saturated nature of nuclear forces) but fails to explain the details of light nuclear binding energy and local fluctuations caused by quantum skin structure. In light nuclei, nuclear properties are dominated more by individual quantum effects than by collective effects, which include the existence of shell structure that greatly influences the stability of light nuclei and pairing effects that govern the interaction between pairs of nucleons with opposite spins, contributing significantly to stability. However, Figure 2 shows that krypton (Kr-80) has the highest binding energy with a value of 8.98 MeV/nucleon, which differs from the reference by Krane (1998) [7], iron (Fe-56), with a value of 8.79 MeV/nucleon [2]. This is a consequence of the inherent limitations of LDM as a macroscopic model. The shift in the stability peak toward larger mass numbers in this study confirms that pure LDM can only capture global trends in binding energy and cannot represent the fine nuclear structure that places iron as the most stable nucleus empirically. These results are consistent with other studies that have found this shift in binding energy [9], [10].

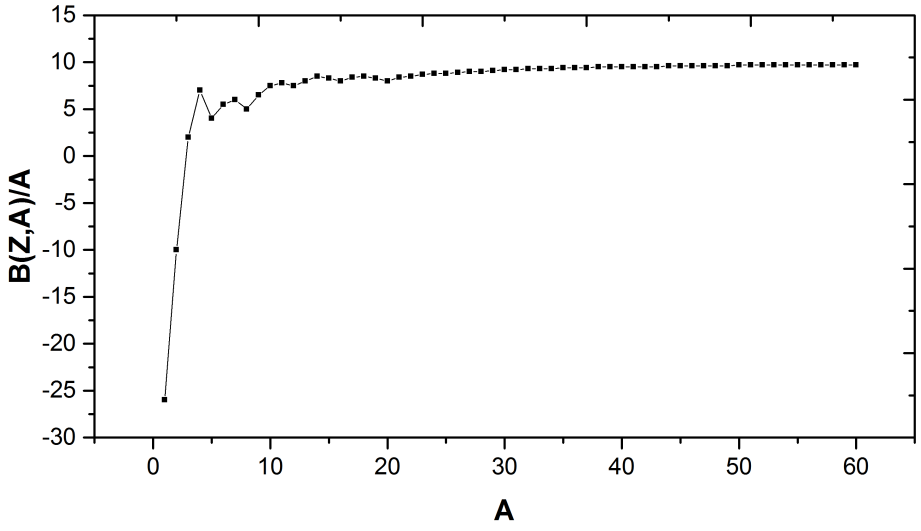

Figure 2: Nuclear binding energy per nucleon using the LDM

4. Conclusion

Calculations of nuclear binding energy using LDM can produce small deviations with mass defect values in medium nuclei, but not in light and heavy nuclei. In heavy nuclei, the number of nucleons is large enough that collective effects are more prominent than the behavior of individual nucleons. The properties of the nucleus (such as binding energy) are better explained by the interaction of all nucleons together (collective effects) rather than by considering only the motion and energy of a single nucleon. In addition, LDM can also predict the binding energy per nucleon. However, the highest binding energy belongs to krypton (Kr-80) with a value of 8.98

MeV/nucleon, which is different from the reference, namely iron (Fe-56), with a value of 8.79 MeV/nucleon.

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